Preface

This monograph aims to provide a concise and comprehensive treatment of the basic theory of algebraic Riccati equations and a description of both the classical and the more advanced algorithms for their solution.

Algebraic Riccati equations are a class of matrix equations which model a variety of different real world problems. Their important role in scientific computing and engineering, together with the richness and the beauty of their theoretical and computational properties, has stimulated a strong interest and an intense research activity over the years.

While the basic theory is established, research concerning the design and analysis of algorithms for solving Riccati equations is still very active and intense, due to the strong demand from a growing number of applications. This results in a great variety of methods and techniques for dealing with algebraic Riccati equations.

The literature on this topic includes some books collecting the basic theoretical properties, the computational methods and the mathematical models from applications in engineering and scientific computing, and several software packages where one can find effective implementation of many algorithms. The available books and software deal mainly with discrete-time and continuous-time algebraic Riccati equations encountered in control theory. The important case of nonsymmetric algebraic Riccati equations is more recent and the results on this subject are scattered across many papers.

As a remedy, a particular emphasis of our work is on nonsymmetric algebraic Riccati equations and, more specifically, on the equations associated with M-matrices due to their relevant importance in the applications in fluid queues, stochastic processes, and transport theory. Another topic treated at length is the analysis of unilateral quadratic matrix equations. In fact, this class is closely related to algebraic Riccati equations and plays also a fundamental role in the solution of stochastic processes associated with queueing models. Concerning algorithms, a large part of the book is devoted to the doubling algorithms, which are very effective in solving algebraic Riccati equations and have not yet been described in any book.

Besides the presentation of the general ideas and the analysis of the tools, we provide a detailed description of all the classical and advanced algorithms for solving algebraic Riccati equations. For the sake of clarity we also give listings of MATLAB codes implementing the algorithms. This has mainly explanatory motivation and does not pretend to be a way to promote advanced and effective software. In fact, we believe that the reader may have a better understanding of the algorithm flow if the implementation code is available. For highly effective software, specifically implemented to solve applications, we refer the reader to the available packages, toolboxes, and libraries such as SLICOT, LYAPACK, RICPAC, HAPACK, MESS, which are cited in the book. However, most of our implementations have been performed by keeping in mind both the goals of efficiency and clarity. In the case where the MATLAB code has only an explanatory motivation, we have added to the function name the suffix "naive." We believe that this collection of functions is the first which implements the classical and the most advanced algorithms for nonsymmetric Riccati equations. The listings of the MATLAB codes can be downloaded at http://riccati.dm.unipi.it/nsare.

Concerning other advanced techniques which are still under development in the literature, we provide a general description to give the flavor of concepts and methods. Moreover, we enrich the presentation with many pointers to the literature which enable the reader to deepen the study if interested in the topic.

The book is addressed to researchers who work in the design and analysis of algorithms and wish to improve or to elaborate and adapt the known techniques to specific problems of interest. It can be also used by practitioners who are solving problems from applications and need a simple explanation of the available algorithms, together with explicit software for their solution. Indeed, this monograph can be of great help to scholars with no expertise in this area who wish to approach this subject from the theoretical or the computational point of view. The book can certainly be used in a semester course on algebraic Riccati equations or as a support in any course in advanced numerical linear algebra and applications.

The book is organized as follows.

In Chapter 1, we introduce the basic definitions concerning algebraic Riccati equations and unilateral quadratic matrix equations, giving their properties and some applications. Tools and concepts from control theory are also recalled.

Chapter 2 concerns theoretical issues related to algebraic Riccati equations. The analysis is subdivided according to discrete-time, continuous-time, nonsymmetric equations and equations associated with M-matrices. Properties of invariant and deflating subspaces and their relationships with the matrix solutions are discussed. Specific attention is addressed to extremal solutions, where the term extremal is referred to the specific semiordering related to the peculiar context. Critical solutions are defined and a way to overcome criticality is introduced. Some transformations mapping the stability region from continuous-time to discrete-time are considered. The class of unilateral quadratic matrix equations and its relationships with Riccati equations are investigated. Finally, the chapter reports some perturbation results on the solutions of matrix equations.

Chapters 3–6 deal with algorithms for solving algebraic Riccati equations. In particular, Chapter 3 deals with classical algorithms. We first consider some linear equations, such as Sylvester, Lyapunov, and Stein equations, whose solution algorithms are important bricks in the design of methods for Riccati equations. Techniques based on computing invariant subspaces, such as the Schur method, are investigated. The Newton iteration is considered together with other functional iterations and iterative refinement. Methods based on the matrix sign function are discussed. The chapter contains some numerical experiments which point out the specific features of the different algorithms.

Chapter 4 deals with methods specifically tailored for the Hamiltonian structure of continuous-time problems. Some Hamiltonian algorithmic machinery and special forms like the Paige–Van Loan (PVL) form and the URV decomposition are introduced. The Hamiltonian QR algorithm, the URV algorithms, and the multishift algorithm are presented.

Chapter 5 is devoted to the class of doubling algorithms. More specifically, the structure-preserving doubling algorithm (SDA) is described in detail in three different versions: SDA-I, SDA-II, and the QR-based form. The cyclic reduction technique (CR) is explained and analyzed. Algorithms based on CR are described for the solution of unilateral quadratic matrix equations and algebraic Riccati equations. Relationships between SDA and CR are pointed out. Particular attention is addressed to nonsymmetric Riccati equations, to equations associated with M-matrices, and to the acceleration techniques. Results of numerical experiments which point out the specific features of the different algorithms are reported.

Chapter 6 provides some pointers to algorithms concerning large-scale problems where the matrix coefficients have some rank structure or are sparse matrices and the solution is well approximated by a low rank matrix. This subject is currently receiving much attention and is still evolving. For this reason, this chapter only reports some ideas and some insights without providing specific implementations. The techniques for dealing with sparse or rank-structured matrix coefficients widely intersect the core numerical linear algebra machinery like the Krylov subspaces methods.

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