

# Index

- ADI, 205
  - Cholesky factor, 199
  - iteration, 196–201
- affine transform, 15, 168, 171–178, 183, 188, 191, 192
  - applied to a pencil, 16, 70, 168
- almost antistabilizing solution, *see* solution
- almost stabilizing solution, *see* solution
- antistabilizing solution, *see* solution
- antistable
  - deflating subspace, *see* deflating subspace
  - eigenvalue, 11
  - invariant subspace, *see* invariant subspace
  - matrix, *see* matrix, 48, 105, 198
- application to
  - fluid queues, 4
  - ladder networks, 7
  - LQ control problem, 5, 7
  - QBD processes, 8
  - transport equation, 4
  - vibration analysis, 9
- Arnoldi process, 202, 214
- backward stability, 29, 107, 131
  - strong, 30, 132, 136, 140
- balancing technique, 92, 142
- Bartels and Stewart algorithm, 84–87, 96, 103, 104, 203
- Brauer’s theorem, 18
  - for matrix polynomials, 19
- breakdown, 101, 140, 149, 150, 153, 176
  - happy (of block Arnoldi), 203
  - of CR, 161–166, 177, 178
  - of multishift algorithm, 142
  - of SDA, 149, 154, 169, 171, 190
  - remediable (block Arnoldi), 203
- c-antistabilizing solution, *see* solution
- c-antistable
  - deflating subspace, *see* deflating subspace
  - eigenvalue, 11
  - invariant subspace, *see* invariant subspace
  - matrix, *see* matrix
- c-splitting, *see* splitting
- c-stabilizable
  - pair, 12, 47, 94, 97
  - triple, 12, 62
- c-stabilizing solution, *see* solution
- c-stable
  - deflating subspace, *see* deflating subspace
  - eigenvalue, 11
  - invariant subspace, *see* invariant subspace
  - matrix, *see* matrix
- canonical factorization, 64, 65
- canonical invariant subspace, 22, 23, 36, 51, 136
- canonical Jordan form, 10, 13, 14, 17, 35, 105
- Cayley shift, 134
- Cayley’s transform, 15–17, 110, 168, 170, 172–174, 177, 178, 180, 183, 186, 188, 191, 192
  - applied to a pencil, 16, 70

- applied to Hamiltonian matrices, 24
  - applied to Hamiltonian pencils, 25
  - generalized, 15
- Cholesky factor ADI, 199
- closed loop matrix, 41, 49, 52
- condensed form
  - $\mathcal{J}$ -tridiagonal, 131, 135
  - Hamiltonian, 134, 135
  - Hamiltonian Hessenberg, 131
  - of a matrix, 126
  - PVL form, 126–128, 135
- controllable
  - pair, 12, 49, 95
  - triple, 12, 62
- convergence, 26–28
  - linear, 27
  - quadratic, 27
  - sublinear, 27
  - superlinear, 27
- CR, 155–167
  - applicability, 161–166
  - breakdown, 161–166, 177, 178
  - convergence, 158–161
  - interplay with SDA, 167
- critical
  - eigenvalue, 11, 12
  - solution, *see* solution, 183, 190
- critical solution, 66
- cyclic reduction, *see* CR
- d-antistable
  - deflating subspace, *see* deflating subspace
  - eigenvalue, 11
  - invariant subspace, *see* invariant subspace
  - matrix, *see* matrix
- d-splitting, *see* splitting
- d-stabilizable
  - pair, 12, 50
  - triple, 12, 62
- d-stable
  - deflating subspace, *see* deflating subspace
- eigenvalue, 11
- invariant subspace, *see* invariant subspace
- matrix, *see* matrix
- solution, *see* solution
- decomposition
  - generalized Schur, 88–90
  - periodic Schur, 137, 139
  - real Schur, 212
  - Schur, 28, 85, 87–89, 92, 103, 212
  - semioordered Schur, 88–90
  - URV, 128–130, 135–137, 142
- defect correction, 102–103
- deflating subspace, 9–11, 33–41, 89, 90, 147, 178
  - antistable, 12
  - c-antistable, 12, 168
  - c-stable, 11, 110, 168
  - d-antistable, 12
  - d-stable, 11, 110, 148, 168
  - graph, 146
  - left, 11
  - stable, 11
  - weakly antistable, 12
  - weakly stable, 12
- detectable pair, 12, 48, 79
- double shift, 55, 57–59, 61, 134, 189
- drift, 37, 38, 45, 51, 54, 57, 77, 89, 99, 104, 114, 115, 117, 118, 170, 177, 188
- dual equation, 36, 43, 44, 51, 64, 69, 71, 73, 157, 169, 173, 178, 179, 182, 183
- elementary matrix, *see* matrix
  - Givens, 219
  - Householder, 218
- exact line search, 96–98
- extremal solution, *see* solution
- factorization
  - canonical, 64, 65
  - full rank, 196
  - LU, 29, 108
  - PLU, 28
  - polynomial, 14, 64

- QR, 106, 133, 134, 154, 211
- QR with column pivoting, 203, 211
- QZ, 91
- thin QR, 203, 211
- UL, 70
- Fréchet
  - derivative, 50, 52, 66, 92, 93, 99, 101, 217
  - differentiable function, 218
- generalized
  - Cayley transform, 15, 24
  - inverse, 95
  - Lyapunov equation, 87
  - Schur decomposition, 89, 90, 92, 212
  - Stein equation, 87
  - Sylvester equation, 87, 101
- Givens
  - rotation, 122, 123, 219, 220
  - symplectic matrix, 123
- graph subspace, *see* invariant subspace
- GTH trick, 29
- Hamiltonian
  - condensed form, 125–131, 134, 135
  - eigenproblem, 121, 126, 131
  - Hessenberg form, 131, 135
  - matrix, 22–25, 39, 105, 109, 122, 123, 127–134, 136, 139–141
    - eigenvalues, 105, 135–136, 183
  - PVL form, 126, 141
  - URV decomposition, 128, 130, 135
  - pencil, 25, 182
    - eigenvalues, 25
  - property, 58
  - QR algorithm, 127, 131–135
  - Schur form, 132, 134, 138–140
- Hessenberg
  - form, 85, 135, 214
  - matrix, 127, 128, 131, 133, 203, 214
- Householder
  - matrix, 122, 218–219
- symplectic matrix, 122, 123
- invariant subspace, 9–11, 33–41, 77, 87–92
  - $\mathcal{J}$ -neutral, 39, 140
  - antistable, 12
  - c-antistable, 12, 21, 23, 36, 38, 105, 137, 138
  - c-stable, 11, 21, 23, 38, 51, 88, 105, 110, 132, 136–138, 140
  - canonical, 22, 23, 36
  - d-antistable, 12, 68
  - d-stable, 11, 68, 180
  - graph, 35, 36, 46, 51, 53
  - Lagrangian, 39, 140
  - left, 11
  - stable, 11
  - weakly antistable, 12, 38, 44
  - weakly stable, 12
- iterative refinement, 103
- $\mathcal{J}$ -tridiagonal form, 131
- Jordan
  - block, 212
  - canonical form, 10, 13, 14, 17, 35, 105, 211
  - chain, 10–14, 17, 18, 22, 23, 34, 212
  - complete chain, 10, 212
- Kleinman's iteration, 94, 205
- Kronecker product, 50–52, 54, 84, 86, 214
- Krylov
  - method, 196, 201–205
  - subspace, 196, 201–204, 213, 214
- Lie group
  - symplectic, 31
- linearizing matrix, 35
- Lyapunov equation, 83–87, 93, 95, 96, 101, 196–199, 201–205
- M-matrix, 3, 5, 29, 37, 38, 42–46, 50, 51, 54–57, 73, 74, 76–77, 85, 98–101, 104, 114–118, 163,

- 164, 168–169, 171, 176, 178, 188, 191, 215–216
- Markov  
 chain, 8, 37  
 continuous-time process, 4  
 QBD process, 8
- matrix  
 antistable, 11, 48, 105, 198  
 block triangular, 70, 88, 89, 148, 211  
 c-antistable, 11  
 c-stable, 11, 85, 88, 94, 95, 105  
 d-antistable, 11  
 d-stable, 11, 63  
 elementary, 218–220  
 elementary symplectic, 122–125, 128, 129, 140  
 function, 15–17, 20, 49, 105, 216  
 geometric mean, 6  
 Givens, *see* Givens  
 Hamiltonian, 22–25, 39, 105, 109, 122, 123, 127–134, 136, 139–141  
 Hessenberg, *see* Hessenberg  
 Householder, *see* Householder  
 M-matrix, *see* M-matrix  
 nonnegative, 2  
 positive, 2  
 quasi-triangular, 84, 132, 133, 136–139, 212  
 sign, 105–110  
 skew-Hamiltonian, 23–24, 127  
 stable, 11, 48  
 symplectic, 22, 24, 123, 131  
 triangular, 87, 89, 92, 95, 115, 127, 128, 132–134, 137–139, 141, 154, 211, 212  
 weakly antistable, 11  
 weakly stable, 11  
 Z-matrix, *see* Z-matrix
- matrix equation, 2–3  
 matrix Laurent polynomial, 217  
 matrix Laurent power series, 217  
 matrix pencil, 10  
 deflating subspace, *see* deflating subspace  
 eigenvalues, 10, 25  
 eigenvalue at infinity, 10  
 Hamiltonian, 25, 182  
 left deflating subspace, 11  
 left similar, 11  
 regular, 10  
 right similar, 11  
 skew-Hamiltonian, 25  
 symplectic, 25
- matrix polynomial, 12–14  
 eigenvalues, 13  
 eigenvalues at infinity, 13  
 quadratic, 13  
 regular, 13
- matrix power series, 217
- maximal solution, *see* solution
- minimal nonnegative solution, *see* solution
- minimal solution, *see* solution
- Moore Penrose inverse, 95
- multiplicity  
 algebraic, 212  
 geometric, 212  
 partial, 10, 15, 22, 36, 47, 49, 51, 57, 58, 60, 183, 212
- multishift algorithm, 127, 135, 140–142  
 breakdown, 142
- Newton increment, 93
- Newton's method, 92–102  
 with exact line search, 96–98
- Newton–Kleinman iteration, 205
- norm, 209  
 Frobenius, 210  
 operator, 209
- null recurrent equation, 37
- numerical cancellation, 29
- observable pair, 12
- Paige–Van Loan form, *see* PVL form
- periodic QR algorithm, 135, 136
- positive recurrent equation, 37, 45
- PVL form, 126–128  
 of a general matrix, 127

- of a Hamiltonian matrix, 126, 128
  - of a skew-Hamiltonian matrix, 127
- QBD process, 8
- QR algorithm
  - Hamiltonian, 127, 131–135
  - periodic, 135, 136
- QR factorization, 211
  - symplectic, 134
  - thin, 211
  - with column pivoting, 211
- quadratic eigenvalue problem, 9
- quasi-triangular matrix, *see* matrix
- rate of convergence, 27
- reachable pair, 12
- regularizable triple, 12, 62
- Ritz value, 198
- Schur
  - algorithm, 89, 91
  - complement, 161, 163–167, 177, 178, 213
  - decomposition, *see* decomposition, 212
  - form, 84–86, 89, 95, 96, 107, 132, 133, 136, 138
  - Hamiltonian form, 132, 134, 138–140
  - method, 88–92
  - periodic decomposition, 137, 139
  - semioordered decomposition, *see* decomposition
  - structured algorithm, 131, 132
- SDA, 146–154
  - applicability, 148, 152
  - breakdown, 149, 154, 169, 171, 190
  - convergence, 153
  - interplay with CR, 167
  - SDA-I, 147–151
  - SDA-II, 151–153
- semiseparable matrix, 134
- Sherman–Woodbury–Morrison formula, 72
- skew-Hamiltonian
  - eigenvalue problem, 127
  - matrix, 23–24, 127
  - pencil, 25
- solution
  - almost c-antistabilizing, 39, 47–48, 52, 168
  - almost c-stabilizing, 39, 47–48, 51, 59, 179, 183
  - almost d-antistabilizing, 49
  - almost d-stabilizing, 41, 49, 52
  - antistabilizing, 44, 47
  - c-antistabilizing, 39, 47–48, 53–55, 57, 99
  - c-stabilizing, 39, 46–48, 51, 57, 59, 61, 88, 105, 107, 110, 179, 180, 182
  - critical, 50–55, 57, 59, 60, 66, 77, 99, 104, 171, 183, 190
  - d-antistabilizing, 49
  - d-stabilizing, 41, 49, 52, 73, 102, 184, 186
  - d-stable, 63–65, 72, 157–161, 170, 177, 190
  - extremal, 41–50
  - maximal, 47, 49
  - minimal, 47, 49
  - minimal nonnegative, 37, 38, 41–46, 50, 51, 54, 55, 77
  - minimal nonnegative solution, 57, 73, 169
    - of a UQME, 8
    - of an M-NARE, 4, 57, 98, 99, 103, 114
  - stabilizing, 39, 47, 50, 79, 88, 93
- spectral radius of a matrix, 3
- splitting, 20–23, 39, 88, 89
  - c-splitting, 21, 23, 25, 36–39, 44, 51, 52, 57, 69, 88, 105, 106, 110, 168, 170, 178, 180
  - d-splitting, 20, 25, 63, 65, 66, 110, 156, 158, 160, 168–170, 174, 178, 186
  - proper, 21, 22, 36, 161, 188
  - strong, 21, 23, 65, 88, 110, 158, 170, 180
  - weak, 21–23, 45, 186, 189
- SSF-I, 147

- SSF–II, 151  
 SSyF, 182  
 stability region, 11, 12, 15, 21  
 stabilizable pair, 12, 48, 79  
 stabilizing solution, *see* solution  
 stable
  - deflating subspace, *see* deflating subspace
  - eigenvalue, 11
  - invariant subspace, *see* invariant subspace
  - matrix, *see* matrix, 48
 standard structured form–I, *see* SSF–I  
 standard structured form–II, *see* SSF–II  
 standard symplectic form, *see* SSyF  
 Stein equation, 83–87, 102, 206  
 strong backward stability, 30, 132, 136, 140  
 structured doubling algorithm–I, *see* SDA–I  
 structured doubling algorithm–II, *see* SDA–II  
 structured doubling algorithms, *see* SDA  
 structured Schur algorithm, 131, 132  
 Sylvester equation, 83–87, 202  
 symmetric Stein equation, 86  
 symplectic
  - elementary matrix, 122–125, 128, 129, 140
  - Givens matrix, 123
  - Householder matrix, 122, 123
  - matrix, 22–24, 131
    - eigenvalues, 25
  - pencil, 25
    - eigenvalues, 25
  - QR factorization, 134
  - similarity, 24
  - unitary matrix, 24
- transient equation, 37
- UL-based transform, 70–75, 174, 178  
 unilateral quadratic matrix equation, *see* UQME
- UQME, 8–9, 62–75, 158–161, 163, 164, 174, 175, 178, 183, 190
- URV
  - algorithms, 136
  - decomposition, *see* decomposition
- Van Loan’s curse, 132, 142
- weakly
  - antistable matrix, 11
  - stable deflating subspace, 12
  - stable invariant subspace, 12
  - antistable deflating subspace, 12
  - antistable invariant subspace, 12
  - stable matrix, 11
- Z-matrix, 57, 74, 164, 176, 215